



Semester One Examination, 2025

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

Section One: Calculator-free

SOLUTIONS

WA student number: In figures

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In words

Circle your Teacher's Name: Ms Lee Mr Liu Mr Mohammadi Mrs Murray

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional
answer booklets used
(if applicable):

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Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	12	12	100	94	65
Total					100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free

35% (51 Marks)

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

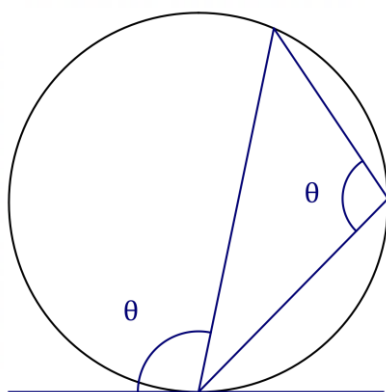
Working time: 50 minutes.

Question 1

(6 marks)

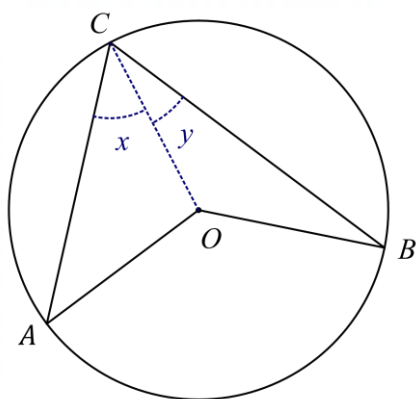
- (a) Use the circle below to illustrate the alternate segment theorem when one pair of equal angles in alternate segments are clearly obtuse. Label this pair of angles with θ .

(2 marks)



Solution
See diagram
Specific behaviours
✓ correct diagram with chords forming obtuse triangle
✓ labels correct pair of equal angles

- (b) In the following diagram, points A, B and C lie on the circle with centre O . Assuming only basic axioms and triangle properties, prove that the size of the angle subtended by arc AB from point O is twice the size of the angle subtended by arc AB from C . (4 marks)



Accept any logical proof.

Solution
The angle subtended by arc AB from point O is $\angle AOB$ and the angle subtended by arc AB from C is $\angle ACB$. Required to prove $\angle AOB = 2\angle ACB$.
Let $\angle OCA = x$ and $\angle OCB = y$ so that $\angle ACB = x + y$.
Since $OA = OB = OC$ then $\triangle OCA$ and $\triangle OCB$ are both isosceles triangles so that $\angle OAC = \angle OCA = x$ and $\angle OBC = \angle OCB = y$.
Hence $\angle AOC = 180^\circ - 2x$ and $\angle BOC = 180^\circ - 2y$
Angle at O :
$\begin{aligned}\angle AOB &= 360^\circ - \angle AOC - \angle BOC \\ &= 360^\circ - (180^\circ - 2x) - (180^\circ - 2y) \\ &= 2x + 2y \\ &= 2\angle ACB \text{ as required}\end{aligned}$
Specific behaviours
✓ defines variables used and obtains expression for $\angle ACB$ ✓ justifies isosceles triangles i.e. must see $OA = OB = AC$ (radius) ✓ obtains expressions for $\angle AOC$ and $\angle BOC$ ✓ obtains expression for $\angle AOB$ and completes proof

Question 2

(7 marks)

- (a) Forces $\vec{F}_1 = \begin{pmatrix} -66 \\ 120 \end{pmatrix}$ newtons, $\vec{F}_2 = \begin{pmatrix} 48 \\ 33 \end{pmatrix}$ newtons and $\vec{F}_3$ are in equilibrium. Determine $\vec{F}_3$.

(2 marks)

Solution
$\vec{F}_3 = - \left[\begin{pmatrix} -66 \\ 120 \end{pmatrix} + \begin{pmatrix} 48 \\ 33 \end{pmatrix} \right]$ $= \begin{pmatrix} 18 \\ -153 \end{pmatrix} \text{ newtons}$
Specific behaviours
✓ indicates correct method ✓ correct force

- (b) Given that unit vectors $\vec{i}$ and $\vec{j}$ are directed due east and due north respectively, express a velocity of 6 metres per second on a bearing of 120° in exact component form.

(2 marks)

Solution
Angle of direction of velocity with x-axis is $90^\circ - 120^\circ = -30^\circ$. $\vec{v} = 6 \begin{pmatrix} \cos -30^\circ \\ \sin -30^\circ \end{pmatrix}$ $= \begin{pmatrix} 3\sqrt{3} \\ -3 \end{pmatrix} \text{ metres per second}$
Specific behaviours
✓ indicates correct expression for at least one component ✓ correct velocity in component form (no penalty for missing units)

0 marks if the components are reversed.

- (c) A particle is moving with constant velocity $\vec{v}$, so that at $t = 3$ it has position vector $\begin{pmatrix} 2.5 \\ -3.5 \end{pmatrix}$ metres and when $t = 13$ it has position vector $\begin{pmatrix} -3.5 \\ 4.5 \end{pmatrix}$ metres, where t is the time in seconds. Determine the speed of the particle.

(3 marks)

Solution
$(13 - 3)\vec{v} = \begin{pmatrix} -3.5 \\ 4.5 \end{pmatrix} - \begin{pmatrix} 2.5 \\ -3.5 \end{pmatrix}$ $10\vec{v} = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$ $\vec{v} = 0.1 \begin{pmatrix} -6 \\ 8 \end{pmatrix}$ $ \vec{v} = 0.1\sqrt{(-6)^2 + 8^2} = 1$ The speed of the particle is 1 metre per second.
Specific behaviours
✓ correct change in displacement ✓ correct velocity ✓ correct speed

Question 3

(9 marks)

(a) Consider the statement:

If n is a square number and if n is a multiple of 2, then n is a multiple of 4.

Write down the inverse of this statement.

(1 mark)

Solution
"If n is not a square number or if n is not a multiple of 2, then n is not a multiple of 4." OR "If n is neither a square number or a multiple of 2, then n is not a multiple of 4."
Specific behaviours
✓ correct statement (the use of "nor" instead of "or" is incorrect)

(b) (i) Prove the following statement is false:

(3 marks)

$\exists$ a real number, x , such that $-x^2 + 4x - 5 \geq 0$

Solution
$x^2 - 4x + 5 \leq 0$ $(x - 2)^2 + 1 \leq 0$ Min Turning point = (2, 1) Minimum value of $x^2 - 4x + 5$ is 1, hence no real number x exists such that $-x^2 + 4x - 5 \geq 0$
Specific behaviours
✓ completes the square ✓ determines the turning point ✓ explains why the statement is false

Alternate Solution
$\Delta = 4^2 - 4(-1)(-5)$ $= -4$ Since $a < 0$ and $\Delta < 0 \Rightarrow \nexists x: x^2 + 4x - 5 > 0$
Specific behaviours
✓ calculates discriminant ✓ states that $a < 0$ and $\Delta < 0$ ✓ explains why the statement is false

(ii) State the range of values of c for which the following statement is true:

(2 marks)

$\exists$ some real numbers, x , such that $-x^2 + 4x - c \geq 0$

Solution
$x^2 - 4x + c \leq 0$ $(x - 2)^2 + c - 4 \leq 0$ Min turning point at (2, $c - 4$) $c - 4 \leq 0$ $c \leq 4$
Specific behaviours
✓ recognises the y -coordinate of turning point is positive or zero, or discriminant is positive or zero ✓ correct range of values

(c) Prove that, if two even counting numbers differ by four, then their squares differ by a multiple of 16.

(3 marks)

Solution
Let $a = 2k$ and $b = 2k + 4$, $k \in \mathbb{N}^+$ $a^2 = 4k^2$ and $b^2 = 4k^2 + 16k + 16$ $b^2 - a^2 = 4k^2 + 16k + 16 - 4k^2$ $= 16(k + 1)$ Therefore the difference of squares is a multiple of 16.
Specific behaviours
✓ defines a and b correctly ✓ expands and simplifies to show multiple of 16 ✓ concluding statement

Question 4

(8 marks)

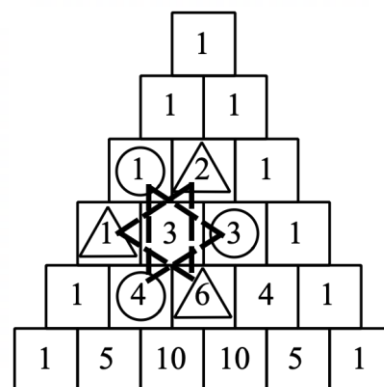
The first six rows of Pascal's triangle are shown on the right.

An example of the 'star' pattern is shown on Pascal's triangle.

In the star pattern we sum the product of the three numbers in the circles, with the product of the three numbers in the triangles, and the number in the middle of the star.

For example, using the highlighted numbers:

$$1 \times 3 \times 4 + 2 \times 1 \times 6 + 3 = 27 = 3^3$$



- (a) State the star pattern that gives 125. (2 marks)

Solution
$1 \times 10 \times 6 + 4 \times 1 \times 15 + 5 = 125$
Specific behaviours
✓ chooses correct values from Pascal's triangle ✓ correct calculation

- (b) (i) Write down an expression in terms of combinations that gives n^3 , where n is defined as row n . (3 marks)

Solution
$\binom{n-1}{0} \binom{n}{2} \binom{n+1}{1} + \binom{n-1}{1} \binom{n}{0} \binom{n+1}{2} + \binom{n}{1} = n^3$
Specific behaviours
✓ first product correct ✓ second product correct ✓ adds nC_1

Alternate Solution
$\binom{n-1}{n-1} \binom{n}{n-2} \binom{n+1}{n} + \binom{n-1}{n-2} \binom{n}{n} \binom{n+1}{n-1} + \binom{n}{n-1} = n^3$
Specific behaviours
✓ first product correct ✓ second product correct ✓ adds nC_1

0 marks if written in terms of n and r .

- (ii) Prove that your expression in part (b)(i) gives n^3 . (3 marks)

Solution
$\begin{aligned} & \binom{n-1}{0} \binom{n}{2} \binom{n+1}{1} + \binom{n-1}{1} \binom{n}{0} \binom{n+1}{2} + \binom{n}{1} = n^3 \\ &= \frac{n!}{2!(2-2)!} (n+1) + (n-1) \frac{(n+1)!}{2!(n-1)!} + n \\ &= \frac{n(n-1)(n+1)}{2} + \frac{n(n-1)(n+1)}{2} + n \\ &= n(n^2 - 1) + n \\ &= n^3 \end{aligned}$
Specific behaviours
✓ substitutes into combination formula (second line) ✓ simplifies factorials (third line) ✓ simplifies to give n^3
NB: Alternate solution at back

Question 5

(7 marks)

- (a) The vertices of parallelogram $EFGH$ lie in alphabetical order on the circumference of a circle. Sketch a diagram of the parallelogram and show that $\angle EFG = 90^\circ$.

(3 marks)

No marks for stating diagonal is diameter and using angles in a semi-circle (?)

Solution

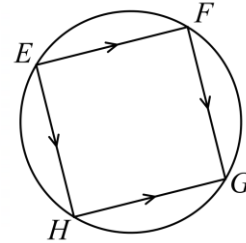
$$\angle EFG = \angle EHG$$

(opposite angles of a parallelogram)

$$\angle EFG + \angle EHG = 180^\circ$$

(opposite angles in cyclic quadrilateral)

$$\text{Hence } 2\angle EFG = 180^\circ \text{ and so } \angle EFG = 90^\circ.$$



Specific behaviours

- ✓ sketch of any quadrilateral in a circle
- ✓ uses one opposite angle theorem
- ✓ uses second opposite angle theorem and deduces required result

- (b) Use a vector method to prove that the diagonals OS and RT of parallelogram $ORST$ intersect at right angles if and only if the parallelogram is a rhombus.

$$\text{Let } \vec{OR} = \vec{r} \text{ and } \vec{OT} = \vec{t}.$$

(4 marks)

Solution

Vectors for diagonals are $\vec{OS} = \vec{t} + \vec{r}$ and $\vec{RT} = \vec{t} - \vec{r}$.

RTP 1: If $ORST$ is a rhombus then the diagonals are perpendicular.

For a rhombus $|\vec{t}| = |\vec{r}|$

$$\begin{aligned} \vec{OS} \cdot \vec{RT} &= (\vec{t} + \vec{r}) \cdot (\vec{t} - \vec{r}) \\ &= \vec{t} \cdot \vec{t} - \vec{t} \cdot \vec{r} + \vec{r} \cdot \vec{t} - \vec{r} \cdot \vec{r} \\ &= \vec{t} \cdot \vec{t} - \vec{r} \cdot \vec{r} \\ &= |\vec{t}|^2 - |\vec{r}|^2 \\ &= 0 \end{aligned}$$

-1 for incorrect notation i.e. no dot between vectors, r^2 instead of $|\vec{r}|^2$ etc.

The dot product of the diagonals is zero, therefore they are perpendicular.

RTP 2: If the diagonals of $ORST$ are perpendicular, then it is a rhombus.

Scalar product of diagonals:

$$\vec{OS} \cdot \vec{RT} = |\vec{t}|^2 - |\vec{r}|^2$$

When diagonals intersect at right angles their scalar product will be zero:

$$\text{Hence } |\vec{t}|^2 - |\vec{r}|^2 = 0 \Rightarrow |\vec{t}|^2 = |\vec{r}|^2 \Rightarrow |\vec{t}| = |\vec{r}|.$$

This result means that the diagonals of a parallelogram will only intersect at right angles if all side lengths are equal ($|\vec{t}| = |\vec{r}|$) i.e. when the parallelogram is a rhombus.

Specific behaviours

- ✓ sets conditions for $ORST$ to be a rhombus i.e. sides of equal length
 - ✓ shows scalar product is zero and concluded diagonals are perpendicular
 - ✓ finds scalar product of diagonals in terms of $\vec{r}$ and $\vec{t}$
 - ✓ shows that for the scalar product to be zero, $|\vec{t}| = |\vec{r}|$ and concludes it must be a rhombus
- NB: If they have proved one direction clearly and alluded or made some attempt to prove back the other way, award 3 out of 4 marks.

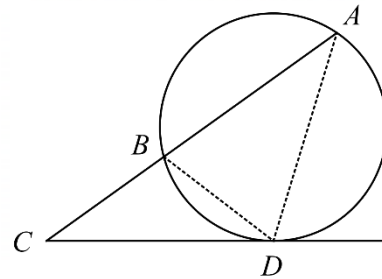
Question 6

(7 marks)

In the diagram, a secant meeting the circle at A and B and a tangent meeting the circle at D are drawn to the circle from C .

SIMPLIFYING QUESTION

Deduct 1 mark overall if any surds not simplified.



- (a) Prove that $CD^2 = CA \times CB$.

(3 marks)

Solution
$\angle ACD = \angle BCD$ (common to $\triangle ACD$ and $\triangle DCB$) $\angle BDC = \angle BAD$ (alternate segment theorem) $\therefore \triangle ACD \sim \triangle DCB$ (AAA)
Using the ratio of corresponding sides: $\frac{CD}{CA} = \frac{CB}{CD} \Rightarrow CD^2 = CA \times CB$
Specific behaviours
✓ shows two pairs of angles are equal ✓ supplies reasons for equal angles and similarity ✓ uses ratio of corresponding sides to derive result

- (b) Given that all dimensions are in centimetres, determine as exact values

- (i) the length of CD when $CB = 15$ and $AB = 9$.

(2 marks)

Solution
$CD^2 = CA \times CB$ $= (15 + 9) \times 15$ $= 2 \times 4 \times 3 \times 3 \times 5$ $CD = 6\sqrt{10} \text{ cm}$
Specific behaviours
✓ substitutes correct values ✓ correct length

- (ii) the length of AC when $CD = 5$ and $AB = 10$.

(2 marks)

Solution
Let $AC = x$, where $x > 0$. $x(x - 10) = 5^2$ $x^2 - 10x = 25$ $(x - 5)^2 = 25 + 25$ $x = 5 \pm 5\sqrt{2}$ Hence $AC = 5 + 5\sqrt{2} \text{ cm}$.
Specific behaviours
✓ indicates correct quadratic equation ✓ correct length and disregard $x < 0$

See next page

Question 7

(7 marks)

Let $\tilde{r} = \begin{pmatrix} -11 \\ -3 \end{pmatrix}$, $\tilde{s} = \begin{pmatrix} -15 \\ 5 \end{pmatrix}$ and $\tilde{t} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}$.

(a) Determine $\tilde{r} + 2\tilde{s} + 3\tilde{t}$.

(2 marks)

Solution
$\tilde{r} + 2\tilde{s} + 3\tilde{t} = \begin{pmatrix} -11 \\ -3 \end{pmatrix} + \begin{pmatrix} -30 \\ 10 \end{pmatrix} + \begin{pmatrix} 12 \\ -12 \end{pmatrix} = \begin{pmatrix} -29 \\ -5 \end{pmatrix}$
Specific behaviours
✓ correct scalar multiples ✓ correct sum

(b) Determine the magnitude of $\tilde{s} - \tilde{r}$.

(2 marks)

Solution
$\tilde{s} - \tilde{r} = \begin{pmatrix} -15 \\ 5 \end{pmatrix} - \begin{pmatrix} -11 \\ -3 \end{pmatrix} = \begin{pmatrix} -4 \\ 8 \end{pmatrix}$
$\left \begin{pmatrix} -4 \\ 8 \end{pmatrix} \right = 4 \left \begin{pmatrix} -1 \\ 2 \end{pmatrix} \right = 4\sqrt{5}$
Specific behaviours
✓ correct difference ✓ correct magnitude

(c) Determine the value of the constant μ and the value of the constant λ when $\tilde{r} = \mu\tilde{s} + \lambda\tilde{t}$.

(3 marks)

Solution
$\begin{pmatrix} -11 \\ -3 \end{pmatrix} = \mu \begin{pmatrix} -15 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -4 \end{pmatrix}$ Comparing coefficients: $\begin{aligned} -11 &= -15\mu + 4\lambda \\ -3 &= 5\mu - 4\lambda \end{aligned}$ Eliminating λ: $-11 - 3 = -15\mu + 5\mu \Rightarrow \mu = 1.4$ Substituting: $-3 = 5(1.4) - 4\lambda \Rightarrow \lambda = 2.5$
Specific behaviours
✓ uses coefficients to form simultaneous equations ✓ correct value of μ ✓ correct value of λ

Accept fractions, including $\frac{14}{10}$

Supplementary page

Question number: _____

Alternate Solution

$$\begin{aligned}
 & \binom{n-1}{n-1} \binom{n}{n-2} \binom{n+1}{n} + \binom{n-1}{n-2} \binom{n}{n} \binom{n+1}{n-1} + \binom{n}{n-1} = n^3 \\
 &= \frac{n!}{(n-2)!(n-(n-2))!} \times \frac{(n+1)!}{n!(n+1-n)!} + \frac{(n-1)!}{(n-2)!(n-1-(n-2))!} \times \frac{(n+1)!}{(n-1)!(n+1-(n-1))!} + \frac{n!}{(n-1)!(n-(n-1))!} \\
 &= \frac{(n+1)!}{(n-2)!2!} + \frac{(n+1)!}{(n-2)!2!} + \frac{n!}{(n-1)!} \\
 &= \frac{(n+1)!}{(n-2)!} + \frac{n!}{(n-1)!} \\
 &= \frac{(n+1)(n)(n-1)(n-2)!}{(n-2)!} + \frac{n(n-1)!}{(n-1)!} \\
 &= (n^2 + n)(n-1) + n \\
 &= n^3 - n^2 + n^2 - n + n \\
 &= n^3
 \end{aligned}$$

Specific behaviours

- ✓ substitutes into combination formula (second line)
- ✓ simplifies factorials
- ✓ simplifies to give n^3

Supplementary page

Question number: _____

